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Semileptonic decays of charmed mesons in the effective action of QCD

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Abstract. Within the framework of phenomenological Lagrangians we construct the effective action of QCD relevant for the study of semileptonic decays of charmed mesons. Hence we evaluate the form factors of $D \to P(0^-)\ell^+\nu_\ell$ at leading order in the $1/N_C$ expansion and, by demanding their QCD-ruled asymptotic behavior, we constrain the couplings of the Lagrangian. The features of the model-independent parameterization of form factors are provided, and their relevance for the analysis of experimental data are pointed out.

1 Introduction

Matrix elements of hadron currents in exclusive processes provide, from a phenomenological point of view, a detailed knowledge on the hadronization mechanisms. Their evaluation, however, is a long-standing problem due to the fact that involves strong interactions in an energy region where perturbative QCD is unreliable. Within this frame, exclusive semileptonic decays of mesons yield the relevant physical system to analyze matrix elements of flavor changing currents.

When only light quark flavors are involved, as in the $K_{\ell 3}$ or $K_{\ell 4}$ processes, the model-independent rigorous framework of chiral perturbation theory (χPT) allows a thorough study to be made that has been proven successful [1]. Semileptonic decays of B mesons, on the other side, can be studied within the heavy quark effective theory (HQET). This last procedure relies in the fact that, the b quark being much heavier than $\Lambda_{\rm QCD}$ (which determines the typical size of hadrons), the light degrees of freedom interact independently of the flavor or spin orientation of the heavy quark. In practice one expands the amplitudes in inverse powers of the heavy quark mass ($\Lambda_{\rm QCD}/M_b$), and the expansion is most suitable for weak decays where heavy flavors are involved, i.e. $b \to c$ [2].

However charmed mesons decay to light flavors and the c quark is much lighter than the b quark; therefore and though the HQET has also been applied to the study of its semileptonic decays [3,4], involving already a rather cumbersome effective action at the next-to-leading order, it is doubtful that perturbative corrections are small enough to provide a thorough result. Another approach involves a mixed framework including HQET and modelizations [5] that, although predictive, rely in ad hoc assumptions not well justified. In addition there is no χPT framework appropriate to perform this task either because the c quark does not belong to its realm. This no-man's-land position of charm has brought about a feeble status in the study of its decays and, in particular, of $D_{\ell 3}$ semileptonic decays we are interested here. Several analyses exist within lattice QCD [6], QCD sum rules [7], and models using phenomenological approaches [8] or quark realizations [9]. Besides, non-leptonic decays of the charmed mesons, that, up to the present, have only been studied in several modelizations such as factorization [10] or chiral realizations [11], rely within these models on semileptonic form factors. Consequently their study is also relevant for those processes.

From an experimental point of view, while branching ratios are rather well measured in both $D \to P\ell^+\nu_\ell$ and $D \to V\ell^+\nu_\ell$ processes[1] [12], the structure of their form factors, relying more on the statistics of events, is loosely known [13]. The E687 and E791 experiments at Fermilab [14–16], BEATRICE at CERN [17], and CLEO at Cornell [18–20] have published their analyses and a further improvement will continue with FOCUS (E831) in the near future, with approximately forty times the previous E687 number of events[2]. Hence form factors in these processes are expected to be thoroughly studied.

Effective actions of the underlying standard model, as χPT or HQET, have become excellent frameworks to carry on analyses of processes of which the relevant physics properties are embodied in phenomenological Lagrangians that contain the proper degrees of freedom and symmetries. The hadronic system we pretend to describe here

[1] If unspecified, P is short for pseudoscalar meson, V for vector meson, and D is short for $D^{+,0}$ or D_s^+. Charge conjugate modes are also implied

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involves charmed mesons and light pseudoscalar mesons or vector resonances. The construction of phenomenological Lagrangians [21,22] gives us a rigorous path to follow when both Goldstone bosons (light pseudoscalar mesons) and matter fields (we include here vector resonances and charmed pseudoscalar mesons) are involved. In addition we will implement this formulation with suitable dynamical assumptions based on the properties of a large number of colors (N_C) [23] and the asymptotic behavior of QCD. These tools have largely been employed together with the construction of phenomenological Lagrangians in order to provide an effective action of the underlying strong interacting field theory in the non-perturbative, resonance dominated, energy region. This procedure has been successfully applied to the construction of the resonance chiral theory [24] providing an excellent basis to parameterize and explore the relevant phenomenology.

Within this frame the goal of this paper is to provide a model-independent QCD-based parameterization of form factors suitable for the analyses of the foreseen new data. To proceed we will construct in Sect. 2 the relevant effective action of QCD for the study of semileptonic decays of charmed mesons. Then we will use this action to evaluate the form factors of the $D \to P\ell^+\nu_\ell$ processes in Sect. 3 and we will impose the constraints that the QCD-ruled asymptotic behavior of form factors demand on the coupling constants, completing therefore the construction of the effective action. This procedure gives a general constrained parameterization of form factors that relies on symmetry properties of the underlying QCD theory without appealing to model-dependent simplifying assumptions. In the following section, Sect. 4, we will comment on the phenomenology and use of our results in order to analyze the experimental data of the $D \to P\ell^+\nu_\ell$ decays. The complete study of the $D \to V\ell^+\nu_\ell$ processes will be carried out in a later publication [25]. In Sect. 5 the relevance of semileptonic processes in determining the couplings of the effective action is pointed out. A comparison of our results with those based on the heavy quark mass expansion will be sketched in Sect. 6 and, finally, Sect. 7 is devoted to our conclusions.

2 The effective action

The present construction of effective field theories of the standard model in different energy regions is based on the theorem put forward by Weinberg in [26] that, schematically, says that the most general Lagrangian containing all terms consistent with the demanded symmetry principles provides general amplitudes with the basic properties of a quantum field theory.

Massless QCD with three flavors has a spontaneously broken chiral symmetry that is manifest in the chiral Lagrangian where Goldstone fields φ_i parameterize the element $u(\varphi)$ of the coset space $G/H \equiv SU(3)_{\rm L} \otimes SU(3)_{\rm R}/SU(3)_V$ given by

$$u(\varphi) = \exp\left(\frac{\mathrm{i}}{\sqrt{2}F}\Pi(\varphi)\right),$$

$$\Pi(\varphi) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix}, \quad (1)$$

where $F \simeq 92.4\,\mathrm{MeV}$ is the pion decay constant. The transformation properties of $u(\varphi)$ under the chiral group G define a non-linear realization of the symmetry through the compensating transformation $h(\varphi) \in SU(3)_V$:

$$u(\varphi) \xrightarrow{G} g_{\rm R} u(\varphi) h(\varphi)^\dagger = h(\varphi) u(\varphi) g_{\rm L}^\dagger,$$
$$g_{\rm L(R)} \in SU(3)_{\rm L(R)}. \quad (2)$$

Non-Goldstone bosons that belong to representations of $SU(3)$ (hence transforming linearly under this group and non-linearly under $SU(3)_{\rm L} \otimes SU(3)_{\rm R}$) can be included in the chiral Lagrangian following [22]. We proceed in turn as follows[3].

(1) For the charmed mesons: Charmed pseudoscalar mesons transform as triplets under $SU(3)$ and we choose the representation

$$D \equiv \begin{pmatrix} \overline{D^0} \\ D^- \\ D_s^- \end{pmatrix}, \quad D \xrightarrow{G} h(\varphi) D, \quad (3)$$

and similarly for charmed resonances $D_{\rm R}$: vector (D_μ^V), axial-vector (D_μ^A) and scalar (D^S). We will introduce different masses for the various triplets of the resonances. Within every triplet we enforce the $SU(3)$ breaking of masses, but we keep $SU(2)$ isospin symmetry.

(2) For the light resonances: We are interested in resonances transforming as octets under $SU(3)$. Following [24] and denoting by $R = V_\mu, A_\mu, S, \ldots$ these octets, the non-linear realization of the chiral group is given by

$$R \xrightarrow{G} h(\varphi) R h(\varphi)^\dagger. \quad (4)$$

The flavor structure of R is analogous to Π in (1). To study the decays we are interested in we will need light vector meson resonances that we introduce as Proca fields.

We would like to establish, using the effective fields above, which is the representation of the generating functional of QCD able to provide matrix elements of charged currents, responsible for semileptonic decays. To define the relation of the effective action with QCD we may consider the effect of external sources J that play the role of auxiliary variables [27]. The link between the underlying and the effective theory is given by the Feynman path integral:

$$\mathrm{e}^{\mathrm{i}\Gamma[J]} = N^{-1} \int [\mathrm{d}\Pi][\mathrm{d}D_{(R)}][\mathrm{d}R]$$
$$\times\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}_{\rm eff}[\Pi, \partial\Pi, D_{(R)}, \partial D_{(R)}, R, \partial R; J, \partial J]}, \quad (5)$$

[3] We do not consider light flavor or charmed singlets in the following. Their inclusion is straightforward with our procedure

where N is the integral evaluated at $J=0$. $\Gamma[J]$ on the left-hand side is the generating functional of the Green functions constructed with the operators of the underlying QCD, while the right-hand side involves the effective field theory. The invariance of the generating functional under gauge transformations of the external sources implements the symmetry properties of the theory.

Therefore the weak interaction is introduced, similarly to the chiral gauge theory framework, through external non-propagating fields. To realize the two weak $SU(2)_{\rm L}$ doublets we now couple the quarks $q=(u,d,s,c)$ to $SU(4)$-valued hermitian external fields $\tilde{\ell}_\mu$, $\tilde{r}_\mu$, $\tilde{s}$ and $\tilde{p}$:

$$\mathcal{L}=\mathcal{L}^{m=0}_{\rm QCD}-m_c\bar{c}c+\frac{1}{2}\bar{q}\gamma^\mu\left[\tilde{\ell}_\mu(1-\gamma_5)+\tilde{r}_\mu(1+\gamma_5)\right]q - \bar{q}(\tilde{s}-{\rm i}\tilde{p}\gamma_5)q, \tag{6}$$

though we will only consider the left and right external sources that are the ones needed to introduce the relevant interaction. Note that in absence of external fields a mass term for the charmed quark c remains.

At the meson level the coupling of external sources requires a $SU(4)$ realization that embeds the two weak $SU(2)_{\rm L}$ doublets into the effective Lagrangian. To proceed we construct a 4×4 matrix involving light flavor and charmed pseudoscalars:

$$\tilde{u}^\dagger_{\rm R}=\begin{pmatrix} u(\varphi) & \frac{\rm i}{\sqrt{2}F_D}u(\varphi)D \\ \frac{\rm i}{\sqrt{2}F_D}D^\dagger & F_D/F \end{pmatrix},$$
$$\tilde{u}_{\rm L}=\begin{pmatrix} u(\varphi) & \frac{\rm i}{\sqrt{2}F_D}D \\ \frac{\rm i}{\sqrt{2}F_D}D^\dagger u(\varphi) & F_D/F \end{pmatrix},$$
$$\widetilde{U}=\tilde{u}^\dagger_{\rm R}\tilde{u}_{\rm L}, \tag{7}$$

and light flavor and charmed resonances:

$$\widetilde{R}=\begin{pmatrix} R & D_{\rm R} \\ D^\dagger_{\rm R} & 0 \end{pmatrix}. \tag{8}$$

However notice that, in accordance with the transformation properties explained above, light and charm flavored pseudoscalar mesons enter with non-linear and linear realizations, respectively. The role of the $SU(4)$ realization in (7) is to help us to find out the implementation of the external sources, in particular the charged current that relates the charm and light meson sector. Therefore, by no means we are implying a seeming chiral realization with four flavors. In (7) F_D is the decay constant of charmed mesons (defined analogously to the $SU(3)$ octet decay constant F that we identify with the decay constant of the pion).

External chiral sources, suitable for the introduction of weak interactions, are coupled through the definition of covariant derivatives on the relevant objects:

$$\begin{aligned}\varDelta_\mu\widetilde{U}&=\partial_\mu\widetilde{U}-{\rm i}\tilde{r}_\mu\widetilde{U}+{\rm i}\widetilde{U}\tilde{\ell}_\mu,\\ \nabla_\mu\widetilde{R}&=\partial_\mu\widetilde{R}+\left[\widetilde{\varGamma}_\mu,\widetilde{R}\right],\end{aligned} \tag{9}$$

with

$$\widetilde{\varGamma}_\mu=\frac{1}{2}\left\{\tilde{u}_{\rm R}\left[\partial_\mu-{\rm i}\tilde{r}_\mu\right]\tilde{u}^\dagger_{\rm R}+\tilde{u}_{\rm L}\left[\partial_\mu-{\rm i}\tilde{\ell}_\mu\right]\tilde{u}^\dagger_{\rm L}\right\}. \tag{10}$$

The right- ($\tilde{r}_\mu$) and left- ($\tilde{\ell}_\mu$) hand external fields are defined as an extension of the $SU(3)$ case:

$$\tilde{r}_\mu=\begin{pmatrix} r_\mu & 0 \\ 0 & \gamma_\mu \end{pmatrix},\quad \tilde{\ell}_\mu=\begin{pmatrix} \ell_\mu & \omega_\mu \\ \omega^\dagger_\mu & \delta_\mu \end{pmatrix}, \tag{11}$$

and their transformation properties are chosen to give the covariant character, under weak gauge transformations, to the derivatives in (9). On the G/H coset space there are two Maurer–Cartan one-forms (left- and right-chiral) related by parity:

$$\begin{aligned}u(\partial_\mu-{\rm i}\ell_\mu)u^\dagger&=\varGamma_\mu+({\rm i}/2)u_\mu\ ,\\ u^\dagger(\partial_m-{\rm i}r_\mu)u&=\varGamma_\mu-({\rm i}/2)u_\mu\ ,\end{aligned} \tag{12}$$

of which the pullback to the space-time space defines the axial vielbein u_μ and the vectorial connection $\varGamma_\mu$. Stepping down to $SU(3)$, the standard right- and left-handed currents are given by

$$\begin{aligned}r_\mu&=eQ\left(A_\mu-\tan\theta_{\rm W}Z_\mu\right),\\ \ell_\mu&=2M_W\sqrt{\frac{G_{\rm F}}{\sqrt{2}}}\begin{pmatrix} 0 & V_{ud}W^\dagger_\mu & V_{us}W^\dagger_\mu \\ V^*_{ud}W_\mu & 0 & 0 \\ V^*_{us}W_\mu & 0 & 0 \end{pmatrix}\\ &\quad+eQA_\mu+e\left[\frac{1}{\sin2\theta_{\rm W}}Q_{\rm L}-Q\tan\theta_{\rm W}\right]Z_\mu,\end{aligned} \tag{13}$$

with $Q=(1/3){\rm diag}(2,-1,-1)$ and $Q_{\rm L}={\rm diag}(1,-1,-1)$. The charmed mesons require a covariant derivative on the $D_{(R)}$ triplets transforming under $SU(3)_{\rm L}\otimes SU(3)_{\rm R}$ as themselves:

$$\nabla_\mu D_{(R)}=\left[\partial_\mu+\varGamma_\mu+\frac{\rm i}{2}(\gamma_\mu+\delta_\mu)\right]D_{(R)}, \tag{14}$$

where $\varGamma_\mu$ has been defined in (12) and the new chiral sources are

$$\begin{aligned}\gamma_\mu&=\frac{2}{3}e\left[A_\mu-\tan\theta_{\rm W}Z_\mu\right],\\ \delta_\mu&=\frac{2}{3}eA_\mu+e\left[\frac{1}{\sin2\theta_{\rm W}}-\frac{2}{3}\tan\theta_{\rm W}\right]Z_\mu.\end{aligned} \tag{15}$$

Finally the left-handed field ω_μ which drives the weak charged current interaction between the charmed and the light sector (as can be seen in (11)) is given by

$$\omega_\mu=2M_W\sqrt{\frac{G_{\rm F}}{\sqrt{2}}}\begin{pmatrix} 0 \\ V^*_{cd} \\ V^*_{cs} \end{pmatrix}W_\mu, \tag{16}$$

that under the chiral group G transforms as

$$\omega_\mu\xrightarrow{G}g_{\rm L}\omega_\mu,\quad g_{\rm L}\in SU(3)_{\rm L}. \tag{17}$$

We would like to emphasize that the electroweak gauge bosons introduced here are not quantized; they behave as classical fields and do not propagate.

With these definitions we can provide the most general phenomenological Lagrangian involving mesons with u, d, s and c quark content and external fields implementing the weak chiral currents of the standard model. However we are interested here in describing $D_{\ell 3}$ decays that are brought about through charged current processes and we will limit ourselves to this case. Hence we design all the relevant $SU(3)_{\rm L}\otimes SU(3)_{\rm R}$ gauge invariant operators. The objects we need to carry on that construction are the effective field realizations in (1), (3) and (4), the covariant derivative $\nabla_\mu D_{(R)}$ in (14), and the external charged source realization ω_μ in (16); all together with their transformation properties under the gauge chiral group. The resulting effective action is

$$\begin{aligned}\mathcal{S}_{\rm eff} &= \int {\rm d}^4x \mathcal{L}_{\rm eff},\\ \mathcal{L}_{\rm eff} &= \mathcal{L}_{\chi{\rm PT}} + \mathcal{L}_{{\rm R}\chi{\rm PT}} + \mathcal{L}_{\rm kin} + \mathcal{L}_D + \mathcal{L}_{D^S}\\ &\quad + \mathcal{L}_{D^V} + \mathcal{L}_{D^A}, \end{aligned} \tag{18}$$

where $\mathcal{L}_{\chi{\rm PT}}$ is the $SU(3)_{\rm L}\otimes SU(3)_{\rm R}$ chiral Lagrangian by Gasser and Leutwyler [27], and $\mathcal{L}_{{\rm R}\chi{\rm PT}}$ is the $SU(3)$ Lagrangian of the resonance chiral theory [24]. $\mathcal{L}_{\rm kin}$ collects all the kinetic and mass terms of the charmed mesons. It also contributes to the interaction Lagrangian through the covariant derivatives. It reads

$$\begin{aligned}\mathcal{L}_{\rm kin} &= (\nabla^\mu D)^\dagger \nabla_\mu D - D^\dagger \mathcal{M}_D D\\ &\quad + (\nabla^\mu D^S)^\dagger \nabla_\mu D^S - (D^S)^\dagger \mathcal{M}_{D^S} D^S\\ &\quad - \frac{1}{2}(D^V_{\mu\nu})^\dagger (D^V)^{\mu\nu} + (D^V_\mu)^\dagger \mathcal{M}_{D^V} (D^V)^\mu\\ &\quad - \frac{1}{2}(D^A_{\mu\nu})^\dagger (D^A)^{\mu\nu} + (D^A_\mu)^\dagger \mathcal{M}_{D^A} (D^A)^\mu\,, \end{aligned} \tag{19}$$

with $D^R_{\mu\nu} = \nabla_\mu D^R_\nu - \nabla_\nu D^R_\mu$, $R = V, A$, and the diagonal mass matrices $\mathcal{M}_{D^{(R)}}$ carry explicit $SU(3)$ breaking. We give here in detail the remaining terms of (18).

(1) Charmed pseudoscalars and light flavored mesons:

$$\begin{aligned}\mathcal{L}_D &= \frac{F_D}{\sqrt{2}}\left[(\nabla^\mu D)^\dagger\, u\omega_\mu + \omega^\dagger_\mu u^\dagger \nabla^\mu D\right]\\ &\quad + {\rm i}\frac{\alpha_1 F}{2\sqrt{2}}\left[D^\dagger u^\mu u\omega_\mu - \omega^\dagger_\mu u^\dagger u^\mu D\right]\\ &\quad + {\rm i}\frac{\alpha_2 m_D^2}{4F}\left[D^\dagger V_\mu u\omega^\mu - \omega^\dagger_\mu u V^\mu D\right]\\ &\quad + {\rm i}\beta_1\left[(\nabla^\mu D)^\dagger V_\mu D - D^\dagger V_\mu \nabla^\mu D\right]. \end{aligned} \tag{20}$$

(2) Charmed scalars, charmed pseudoscalars and light flavored mesons:

$$\begin{aligned}\mathcal{L}_{D^S} &= {\rm i}F_{D^S}\left[\left(\nabla^\mu D^S\right)^\dagger u\omega_\mu - \omega^\dagger_\mu u^\dagger \nabla^\mu D^S\right]\\ &\quad + \beta_2\left[D^\dagger u_\mu \nabla^\mu D^S + \left(\nabla^\mu D^S\right)^\dagger u_\mu D\right]\\ &\quad + \beta_3\left[(\nabla^\mu D)^\dagger u_\mu D^S + D^{S\dagger} u_\mu \nabla^\mu D\right]. \end{aligned} \tag{21}$$

(3) Charmed vectors, charmed pseudoscalars and light flavored mesons:

$$\begin{aligned}\mathcal{L}_{D^V} &= \frac{F_{D^V} m_{D^V}}{2\sqrt{2}}\left[D^\dagger_\mu u\omega^\mu + \omega^\dagger_\mu u^\dagger D^\mu\right]\\ &\quad + {\rm i}\beta_4 m_{D^V}\left[D^\dagger_\mu u^\mu D - D^\dagger u_\mu D^\mu\right]\\ &\quad + \frac{\beta_\varepsilon}{2m_D}\varepsilon_{\mu\nu\alpha\beta}\left[D^\dagger V^{\mu\nu}\nabla^\alpha D^\beta + \left(\nabla^\alpha D^\beta\right)^\dagger V^{\mu\nu} D\right]. \end{aligned} \tag{22}$$

(4) Charmed axial-vectors, charmed pseudoscalars and light flavored mesons:

$$\begin{aligned}\mathcal{L}_{D^A} &= \frac{F_{D^A} m_{D^A}}{2\sqrt{2}}\left[D^{A\dagger}_\mu u\omega^\mu + \omega^\dagger_\mu u^\dagger D^{A\,\mu}\right]\\ &\quad + {\rm i}\beta_5 m_{D^A}\left[D^{A\dagger}_\mu V^\mu D - D^\dagger V^\mu D^A_\mu\right]. \end{aligned} \tag{23}$$

Here we have used $V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$, $\nabla_\mu V_\nu = \partial_\mu V_\nu + [\Gamma_\mu, V_\nu]$, and m_{D^i}, $i = S, V, A$ are typical mass scales for every J^P introduced to define the dimensionless couplings α_i, β_i and β_ε. All together we have 12 a priori unknown coefficients: the decay constants F_D, F_{D^S}, F_{D^V} and F_{D^A}, and the couplings α_i, $i = 1, 2$, β_ε and β_j, $j = 1, 2, 3, 4, 5$. Some information on the masses is known and we may consider them as input in our study. The interacting effective Lagrangian $\mathcal{L}_{\rm eff}$ provides a physically grounded parameterization of the $D \to P\ell^+\nu_\ell$ and $D \to V\ell^+\nu_\ell$ processes without model-dependent assumptions and hence it is a suitable basis for the analyses of the experimental data. It is clear, though, that the number of unknown couplings seems to undertone our task. In the construction of $\mathcal{S}_{\rm eff}$ we have exploited the rigorous constraints that symmetries of the underlying QCD enforce on its effective field theory. However, symmetries give us the structure of the operators but do not tell us anything about their coupling constants. In the next section we will come back to this point.

A thorough explanation of the features and properties of the charm pieces of $\mathcal{L}_{\rm eff}$ is now required. The effective action of QCD in this energy region, as any effective field theory, has an infinite number of pieces. We have collected only those that contribute to the $D_{\ell 3}$ processes with the fewer number of derivatives. This is so because, even if the included vertices can give also a contribution to the $D_{\ell 4}$ processes, for example, many other terms in the full effective action also contribute and should be taken into account. Though the chiral structure of the couplings might be suspect for the production of two or more non-soft light pseudoscalars, it should be correct for the vertices under consideration where only one light pseudoscalar is involved. This statement follows because, on one side, fields are not observables and hence physics does not depend on the field realization. In addition hadron effective fields have very limited freedom in the structure of their couplings and light pseudoscalars only can saturate Lorentz indices through derivatives. Moreover the requirement of chiral symmetry not only enforces the proper matching of the effective action at low energies. Although chiral dynamics is often thought of as imposing constraints only on low momentum processes, it also affects even the high

energy behavior, a result worked out from analyticity [28]. As a consequence, the structure of the couplings in our effective action $\mathcal{S}_{\rm eff}$ is the most general one available for two- and three-legs vertices and, consequently, they should be able to describe both soft and hard outgoing light pseudoscalar mesons. A similar situation happens in the acknowledged resonance chiral theory where, for example, the $a_1(1260) \to \pi\gamma$ process is described along the same lines we use in our effective action. We conclude that the structure of the vertices in $\mathcal{S}_{\rm eff}$ is the appropriate one to deal with $D_{\ell 3}$ processes in the whole energy range.

Note that, contrarily to previous phenomenological Lagrangian approaches in [3,4], the construction of the effective action of QCD that we have carried out does not rely on the heaviness of the charm quark but on the feature that non-Goldstone bosons belonging to irreducible representations of $SU(N_F)$ can consistently be introduced in an effective Lagrangian with the proper QCD symmetries [21,22]. Besides, HQET is an excellent perturbative framework to start with in the B meson sector where inverse mass corrections are reasonably very small and provide the relevant breaking to the heavy quark symmetry limit of QCD. Though rather massive it is not clear that this effective theory can be applied to the charm sector and, in any case, perturbative corrections would be much bigger, spoiling the convergence.

3 Form factors in $D \to P\ell^+\nu_\ell$ decays

$D_{\ell 3}$ processes with a pseudoscalar P in the final state are driven by a hadronic vector H_μ defined through the amplitude of the decay

$$M\left(D \to P\ell^+\nu_\ell\right) = -\frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM}\, \overline{u}_\nu \gamma^\mu (1-\gamma_5) v_\ell H_\mu, \quad (24)$$

and that corresponds to the matrix element of the relevant vector hadronic current driven by the W_μ field:

$$\mathcal{V}_\mu = 2\frac{\delta \mathcal{S}_{\rm eff}}{\delta \omega^\mu}\bigg|_{J=0}, \quad (25)$$

because only this current contributes to the processes under consideration. In (25) J is short for all external sources. Hence we obtain H_μ by differentiating the generating functional of our effective action. Its Lorentz decomposition is written out in terms of the two independent hadron four-momenta in $D(p_D) \to P(p)\ell^+\nu_\ell$:

$$\begin{aligned} H_\mu &\doteq \langle P(p)|\mathcal{V}_\mu \mathrm{e}^{\mathrm{i}\mathcal{S}_{\rm eff}[J=0]}|D(p_D)\rangle \\ &= f_+(q^2)(p_D+p)_\mu + f_-(q^2)(p_D-p)_\mu, \end{aligned} \quad (26)$$

with $q^2 = (p_D - p)^2$, thus introducing the two form factors associated to the process. The $\exp(\mathrm{i}\mathcal{S}_{\rm eff}[J=0])$ term in the definition of H_μ reminds us that the matrix element of the current has to be evaluated in the presence of strong interactions. In terms of these form factors the spectrum of the semileptonic decay is given by

$$\begin{aligned} &\frac{\mathrm{d}\Gamma(D \to P\ell^+\nu_\ell)}{\mathrm{d}q^2} \\ &= \frac{G_{\rm F}^2 |V_{\rm CKM}|^2}{384\pi^3 m_D^3} \frac{\sqrt{\lambda(q^2, m_D^2, m_P^2)}}{q^6} (q^2 - m_\ell^2)^2 \\ &\times \Big\{ |f_+(q^2)|^2 \\ &\times \left[(2q^2 + m_\ell^2)\lambda(q^2, m_D^2, m_P^2) + 3m_\ell^2 (m_D^2 - m_P^2)^2\right] \\ &+ 3q^2 m_\ell^2 \\ &\times \left[2\mathrm{Re}(f_+(q^2) f_-^*(q^2))(m_D^2 - m_P^2) + |f_-(q^2)|^2 q^2\right] \Big\}, \end{aligned} \quad (27)$$

where $\lambda(a,b,c) = (a+b-c)^2 - 4ab$ and, though not explicitly stated, $f_\pm(q^2) \equiv f_\pm(q^2)[D,P]$. When $m_\ell = 0$ the spectrum only depends on the $f_+(q^2)$ form factor and therefore the dependence on $f_-(q^2)$ is suppressed, particularly for $\ell = e$.

3.1 Form factors from the effective action in the $N_C \to \infty$ limit

It has been widely emphasized [29] that large number of colors properties of QCD provide a guiding tool about basic features of the strong interaction dynamics and, therefore, we intend to perform the evaluation of the semileptonic form factors, defined above, at the leading order in the $1/N_C$ expansion. To proceed we recall that the hadron matrix element H_μ in (26) is related with the three-currents Green function

$$\mathcal{G}_\mu \equiv \langle 0|\mathcal{P}_D(x)\mathcal{P}_P(y)\mathcal{V}_\mu(z)|0\rangle$$

where $\mathcal{P}_D(x)$ and $\mathcal{P}_P(x)$ are the pseudoscalar sources with charm and light-quark quantum numbers, respectively, and $\mathcal{V}_\mu(x)$ is the vector hadronic current in (25). The $1/N_C$ expansion gives precise information on the Green functions of QCD currents [23]. In the $N_C \to \infty$ limit the three-point function $\mathcal{G}_\mu$ is a sum of tree diagrams, with free field propagators and local vertices. These diagrams are of two types: in the first, one of the currents creates two mesons, each of which is absorbed by the remaining currents (see Fig. 1a), in the second each current creates one meson, and the three mesons combine in a local vertex (see Fig. 1b). Moreover one has to sum over all the possible propagating mesons. At the next-to-leading order in the $1/N_C$ expansion meson loops have to be taken into account.

Coming back to our matrix element H_μ we see that the pseudoscalar sources, creating the initial and final state mesons, are fixed and, in consequence, the $N_C \to \infty$ limit tells us that we should consider the diagrams in Fig. 1, where in (b) we must sum over the infinite intermediate single resonances with $\mathcal{V}_\mu$ quantum numbers and with local couplings to D and P mesons. Within this approach and, as in the resonance chiral theory, we will assume that nearby resonances provide most of the dynamics of the interaction; heavier resonance contributions

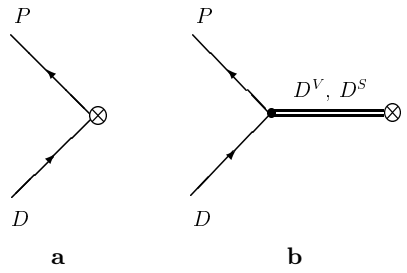

Fig. 1a,b. Tree-level contributions to H_μ. The crossed circle indicates the external source insertion $\mathcal{V}_\mu$ and the black dot is a strong interacting vertex. D^V and D^S are short for charmed vector and scalar resonances, respectively

being suppressed because of their mass[4]. Hence to proceed we evaluate the matrix element in (26) by approaching $\exp(\mathrm{i}\mathcal{S}_{\rm eff}) \sim 1 + \mathrm{i}\mathcal{S}_{\rm eff}$. As we can see the whole strong interaction, at this leading order, is reduced to the contribution in Fig. 1b and it is mediated by charmed resonances. We obtain the following results:

$$M(D \to P\ell^+\nu_\ell) = -\frac{G_{\rm F}}{\sqrt{2}}\overline{u}_\nu\gamma^\mu(1-\gamma_5)v_\ell \times a(D,P)\cdot\Big[f_+(q^2)[D,P](p_D+p)_\mu + f_-(q^2)[D,P](p_D-p)_\mu\Big], \quad (28)$$

where $a(D,P)$ includes Clebsch–Gordan and Kobayashi–Maskawa couplings:

$$a(D^0,\pi^-) = -\sqrt{2}a(D^+,\pi^0) = a(D_s^+,K^0) = V_{cd}^*,$$
$$a(D^+,\overline{K^0}) = a(D^0,K^-) = V_{cs}^*. \quad (29)$$

The form factors are given by

$$f_+(q^2)[D,P] = \frac{1}{2}\left[\frac{F_D}{F} + \alpha_1 - \beta_4\frac{F_{D^V}}{F}\cdot\frac{m_{D^V}^2}{q^2-(M_V[D,P])^2}\right],$$
$$f_-(q^2)[D,P] = \frac{1}{2}\left[\frac{F_D}{F} - \alpha_1 + 2\sqrt{2}\frac{F_{D^S}}{F}\cdot\frac{(\beta_3-\beta_2)(m_D^2+m_P^2-q^2)+2\beta_2 m_P^2}{q^2-(M_S[D,P])^2} + \beta_4\frac{F_{D^V}}{F}\cdot\frac{m_{D^V}^2}{(M_V[D,P])^2} \times \frac{m_D^2-m_P^2-q^2+(M_V[D,P])^2}{q^2-(M_V[D,P])^2}\right]. \quad (30)$$

The dependence on D and P in the form factors reduces to the masses m_D, m_P of the decaying and outgoing hadron, respectively, and $M_V[D,P]$, $M_S[D,P]$ appearing in the propagators in (30). For the different channels we have

$$M_V[D^+,\pi^0] = M_V[D^0,\pi^-] = M_V[D_s^+,K^0] = m_{D^V},$$
$$M_V[D^+,\overline{K^0}] = M_V[D^0,K^-] = m_{D_s^V},$$
$$M_S[D^+,\pi^0] = M_S[D^0,\pi^-] = M_S[D_s^+,K^0] = m_{D^S},$$
$$M_S[D^+,\overline{K^0}] = M_S[D^0,K^-] = m_{D_s^S}, \quad (31)$$

where the notation for the masses is self-explanatory. From the observed spectrum of the charmed mesons [12], D^V would correspond to $D^*(2010)^\pm$, while D_s^V corresponds to D_s^*. The scalar charmed resonances D^S and D_s^S still have not been observed.

It is well known that $f_-(q^2)$ should vanish if the $SU(4)_F$ symmetry is exact due to the conservation of the vector current contributing to the matrix element in (26). An inspection of our result for $f_-(q^2)$ shows that to get that vanishing result it is not enough to enforce $m_D = m_P$ and $F_D = F$ and, therefore, the couplings of our effective action are not independent from each other in the $SU(4)_F$ limit. This is not surprising because the construction of our effective action $\mathcal{S}_{\rm eff}$ was concerned with symmetry requirements from $SU(3)$ where the strong chiral realization lives and charmed mesons were introduced in a different footing, as it should. It is more instructive, though, to leave this discussion to a later stage and we will come back to it.

A next-to-leading evaluation in the $1/N_C$ expansion would provide, typically, a 30% correction on our final results for $N_C = 3$, although in other applications in resonance chiral theory these are effectively smaller. In any case our approach would be good enough for the analysis of present and foreseen experimental results. The computation of next-to-leading contributions is not feasible at the moment because we would need to consider the effective action at one loop, a non-trivial task beyond the scope of this work.

3.2 QCD-ruled asymptotic behavior of form factors

The results that we have obtained for the $D_{\ell 3}$ form factors in (30) are a consequence of the symmetry requirements enforced by QCD on our effective action $\mathcal{S}_{\rm eff}$. As commented on at the end of Sect. 2, though, symmetries do not constrain the coupling constants of $\mathcal{S}_{\rm eff}$ and, consequently, further insight is needed. To proceed we recall the basic features of effective field theory construction. Essentially this is an ongoing procedure from the high energy scale to the energy region of interest where, in the stepping down, heavier degrees of freedom are integrated out through an evolution process driven by both the renormalization group and matching at the masses of heavier particles, when these decouple. We do not explain in detail the construction [30] but recall two relevant conclusions for our work. First of all, when a heavy particle of mass M is integrated out, what results is a non-local action. A later power expansion in p/M (p is a typical momentum of the process) provides the final local non-renormalizable effective action with derivative couplings, as our $\mathcal{S}_{\rm eff}$. This already tells us that the couplings in the effective action are going to be suppressed by the masses

[4] In addition notice that only a single triplet of vector resonances with the appropriate quantum numbers is known, and none of scalar resonances [12]

of heavier degrees of freedom not present in our action. The second conclusion of this procedure is that only the short-distance information is incorporated into the coupling constants of the effective Lagrangian [30]. This is a powerful statement because, though we do not know how to evolve from QCD down to the hadron level, it means that we can, and should, constrain the couplings in accordance with the high energy behavior of the theory. And this indeed we know, because asymptotic freedom provides a valid perturbative treatment of QCD at high energies.

To proceed we will study the asymptotic behavior ($q^2 \to \infty$) of the form factors of the currents, endowing consequently, relations between the unknown couplings of $\mathcal{S}_{\rm eff}$. The restrictions on the semileptonic form factors involving pseudoscalars imposed by their asymptotic behavior ruled by QCD were already worked out a time ago by Bourrely, Machet and De Rafael [31].

As we have said above $D_{\ell 3}$ decays with one pseudoscalar in the hadronic final state are driven by the vector current $\mathcal{V}_\mu$. Then the form factors are related with the spectral functions associated to the vector two-point function[5]:

$$\begin{aligned}\Pi_{\mu\nu} &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T(\mathcal{V}_\mu(x)\mathcal{V}_\nu^\dagger(0))|0\rangle \\ &= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_1(q^2) + q_\mu q_\nu \Pi_0(q^2), \quad (32)\end{aligned}$$

that are defined by

$$\begin{aligned}&-(g_{\mu\nu}q^2 - q_\mu q_\nu)\mathrm{Im}\Pi_1(q^2) + q_\mu q_\nu \mathrm{Im}\Pi_0(q^2) \quad (33)\\ &= \frac{1}{2}\sum_\gamma \int \mathrm{d}\rho_\gamma (2\pi)^4 \delta^{(4)}(q - p_\gamma)\langle 0|\mathcal{V}_\mu(0)|\gamma\rangle\langle\gamma|\mathcal{V}_\nu^\dagger(0)|0\rangle,\end{aligned}$$

where the summation is extended to all possible hadron states γ with appropriate quantum numbers, and the integration is carried out over the allowed phase space of those states. In (32) Π_1 corresponds to the contributions of the $J^P = 1^-$ quantum numbers and Π_0 to those of $J^P = 0^+$. Between the infinite number of intermediate contributions there is the one given by the semileptonic matrix elements of $D \to P\ell^+\nu_\ell$ given by (26) that we now write, not including the exponential of the effective action explicitly, as

$$\begin{aligned}&\langle 0|\mathcal{V}_\mu(0)|D(p_D)\overline{P}(-p)\rangle \\ &= \eta\left[\left(p_D - \frac{p_D\cdot q}{q^2}q\right)_\mu F_1(q^2) + \frac{q_\mu}{q^2}F_0(q^2)\right], \quad (34)\end{aligned}$$

where η is a Clebsch–Gordan coefficient and the new form factors, defined for convenience in the following discussion, can be related with $f_+(q^2)$ and $f_-(q^2)$ through

$$\begin{aligned}F_1(q^2) &= 2f_+(q^2),\\ F_0(q^2) &= (m_D^2 - m_P^2)f_+(q^2) + q^2 f_-(q^2). \quad (35)\end{aligned}$$

They correspond to 1^- and 0^+ contributions, respectively. Positivity of the spectral functions demands that every contribution of the $|\gamma\rangle$ intermediate states adds up and, therefore, the two-pseudoscalar $|D\overline{P}\rangle$ state in the unitarity relation in (33) is just one of the infinite possible contributions to the spectral functions, to which it provides a lower bound. Performing the phase space integration we obtain

$$\begin{aligned}\mathrm{Im}\Pi_1(q^2) &\geq \frac{\eta^2}{192\pi}\sqrt{\left(1 - \frac{Q_0^2}{q^2}\right)^3\left(1 - \frac{Q_1^2}{q^2}\right)^3}\\ &\quad\times |F_1(q^2)|^2\theta(q^2 - Q_0^2),\\ \mathrm{Im}\Pi_0(q^2) &\geq \frac{\eta^2}{16\pi}\sqrt{\left(1 - \frac{Q_0^2}{q^2}\right)\left(1 - \frac{Q_1^2}{q^2}\right)}\\ &\quad\times \frac{|F_0(q^2)|^2}{q^4}\theta(q^2 - Q_0^2), \quad (36)\end{aligned}$$

where $Q_0^2 = (m_D + m_P)^2$ and $Q_1^2 = (m_D - m_P)^2$.

Perturbative QCD at leading order [32] determines that

$$\begin{aligned}\mathrm{Im}\Pi_1(q^2) &\overset{q^2\to\infty}{\longrightarrow} \frac{1}{4\pi},\\ \mathrm{Im}\Pi_0(q^2) &\overset{q^2\to\infty}{\longrightarrow} 0, \quad (37)\end{aligned}$$

and therefore, heuristically, one would expect that in the asymptotic regime every one of the infinite positive contributions to the spectral function vanishes. This is clearly true for the $J = 0$ spectral function and a reasonable guess for the $J = 1$ vector function, expecting that the sum of the infinite vanishing contributions gives a non-zero finite constant result. Accordingly, from (36) and (37), we demand that the conditions

$$\begin{aligned}F_1(q^2) &\overset{q^2\to\infty}{\longrightarrow} 0,\\ F_0(q^2) &\overset{q^2\to\infty}{\longrightarrow} \text{constant} \quad (38)\end{aligned}$$

are fulfilled. In fact we could also choose that[6] $F_0(q^2) \to 0$ as $q^2 \to \infty$ but, while this is a mandatory guess for $F_1(q^2)$, in the $J = 0$ form factor this would be a stronger condition that is not necessary, according to the heuristic discussion above. From (36) we see that a constant asymptotic behavior is enough, and we keep to this softer assumption. Nevertheless in both cases the constraints on the $f_+(q^2)$ and $f_-(q^2)$ form factors are the same. Using (35) we note that both $f_+(q^2)$ and $f_-(q^2)$ should vanish in the $q^2 \to \infty$ limit. Coming back to (30) we get the following relations between the couplings of the effective action $\mathcal{S}_{\rm eff}$:

$$\begin{aligned}\frac{F_D}{F} + \alpha_1 &= 0,\\ 1 - \sqrt{2}\frac{F_{D^S}}{F_D}(\beta_3 - \beta_2) - \frac{\beta_4}{2}\frac{F_{D^V}}{F_D}\frac{m_{D^V}^2}{(M_V[D,P])^2} &= 0. \quad (39)\end{aligned}$$

[5] The relevant flavor indices of the currents for every process should be understood

[6] We comment later on the consequences of this stronger constraint

Carrying these relations over to the expressions in (30) we get the final parameterization of the form factors in $D \to P\ell^+\nu_\ell$:

$$\begin{aligned} f_+(q^2)[D,P] &= \frac{\Omega[D,P]}{1-\dfrac{q^2}{(M_V[D,P])^2}}, \\ f_-(q^2)[D,P] &= \frac{m_P^2-m_D^2}{(M_V[D,P])^2} f_+(q^2)[D,P] \\ &\quad + \frac{\Lambda[D,P]}{1-\dfrac{q^2}{(M_S[D,P])^2}}, \end{aligned} \tag{40}$$

where

$$\begin{aligned} \Omega[D,P] &= \frac{\beta_4}{2}\frac{F_{D^V}}{F}\frac{m_{D^V}^2}{(M_V[D,P])^2}, \\ \Lambda[D,P] &= \left(\frac{F_D}{F} - \Omega[D,P]\right) \\ &\quad \times \left(1 - \frac{m_D^2}{(M_S[D,P])^2} - \frac{m_P^2}{(M_S[D,P])^2}\right) \\ &\quad - 2\sqrt{2}\frac{F_{D^S}}{F}\beta_2\frac{m_P^2}{(M_S[D,P])^2}\ . \end{aligned} \tag{41}$$

These are our main results, and (40) shows the simplest parameterization of $f_+(q^2)$ and $f_-(q^2)$ consistent with QCD constraints and saturation by resonances. It is interesting to note that while our result for $f_+(q^2)$ coincides with the phenomenological one-pole dominance approach shared by other theoretical studies, $f_-(q^2)$ gets a two-pole structure coming from vector and scalar resonances. This feature brings about into the 0^+ scalar form factor $F_0(q^2)$, (35), the presence of a local non-resonant contribution in addition to the one-pole scalar meson resonance. That local piece is induced by the $J^P = 0^+$ time-like polarization of the vector meson, through the cancellation of the vector resonance pole introduced by the $f_+(q^2)$ term in $F_0(q^2)$.

Our discussion above relies on the high energy behavior of the form factors in (38). As commented on, strictly speaking, QCD enforces a constant (not necessarily vanishing) high energy behavior for $F_0(q^2)$. However, studies [33] that assume factorization at high q^2 and some common lore physics intuition would demand the stronger $F_0(q^2) \overset{q^2\to\infty}{\longrightarrow} 0$ condition. Hence $f_-(q^2)|_{q^2\to\infty}$ would vanish at least as $1/q^4$, requiring, consequently, a pure double pole structure. This would enforce an extra condition on the couplings of the effective Lagrangian, namely,

$$\Lambda[D,P] = \frac{m_D^2-m_P^2}{(M_S[D,P])^2}\Omega[D,P].$$

We call $\Lambda[D,P]_{FACT}$ this value for $\Lambda[D,P]$. The experimental measurement of the $f_-(0)$ would provide, in consequence, relevant information on the QCD structure of the form factors.

As commented on above, in the $SU(4)_F$ limit $f_-(q^2)$ should vanish. We observe that this constraint determines relations between the couplings that are only valid in that limit. Hence we get $\Lambda[D,P]|_{SU(4)} = 0$, which provides a relation between the couplings in this limit. However it is clear that $N_F = 4$ flavor symmetry is badly broken and therefore this condition should not be taken seriously.

Pion pole dominance and $SU(2)_{\rm L} \otimes SU(2)_{\rm R}$ current algebra provide the Callan–Treiman relation between the $K_{\ell 3}$ from factors and the decay constant of the kaon, F_K, which drives the $K_{\ell 2}$ decays [34]: $f_+^{K\pi}(m_K^2) + f_-^{K\pi}(m_K^2) = F_K/F$, in the vanishing pion mass limit. In our case a direct evaluation, from (40) and (41), gives

$$\begin{aligned} &\left.\frac{F_0(m_D^2-m_P^2)}{m_D^2-m_P^2}\right|_{[D,P]} \\ &= f_+(m_D^2-m_P^2) + f_-(m_D^2-m_P^2)|_{[D,P]} \\ &= \frac{F_D}{F} - 2\frac{m_P^2}{(M_S[D,P])^2 - m_D^2 + m_P^2} \\ &\quad \times \left[\frac{F_D}{F} + \sqrt{2}\frac{F_{D^S}}{F}\beta_2 - \Omega[D,P]\right]. \end{aligned} \tag{42}$$

Note that the evaluation point, $q^2 = m_D^2 - m_P^2$, is outside the physical region. In the $SU(3)$ chiral limit $m_P = 0$ and we have $f_+(m_D^2) + f_-(m_D^2)|^{\chi}_{[D,P]} = F_D/F$ as the Callan–Treiman relation entails when applied to the four flavor case. Although the $m_P = 0$, $P = \pi, K$, limit in (42) seems affordable, nothing can be said about the size of the correction because our lack of knowledge on the couplings. However notice that a strong cancellation in the denominator of that term, $(M_S[D,P])^2 - m_D^2 + m_P^2$, if charmed scalar resonances are near, could provide a sizeable contribution.

4 Phenomenology of $D \to P\ell^+\nu_\ell$

As we said in Sect. 1 the FOCUS experiment (E831) at Fermilab is foreseen to provide, in the near future, a thorough study of semileptonic form factors of the charmed mesons. Up to now several observables have been measured with rather good accuracy [14–20] but an exhaustive study of the q^2 behavior of the form factors, even the dominant one $f_+(q^2)$, is still missing.

The exclusive channels studied up to now are the Cabibbo-favored ones $D^+ \to \overline{K^0}\ell^+\nu_\ell$, $D^0 \to K^-\mu^+\nu_\mu$, and the Cabibbo-suppressed $D^+ \to \pi^0\ell^+\nu_\ell$ and $D^0 \to \pi^-e^+\nu_e$, of which the branching ratios are measured reasonably well. The study of the q^2 structure of their form factors, however, is much poorer. Notwithstanding, the E687 experiment has published reasonable spectra for $D^0 \to K^-\mu^+\nu_\mu$ [16] though we have been advised[7] that they are not corrected for background, resolution and acceptance effects and, consequently, should not be used to analyze theoretical form factors.

[7] Private communication received from Will Johns

Table 1. Experimental values for $|f_+(0)|$ and m_+ from $D^0 \to K^-\ell^+\nu_\ell$ decays

Experiment	$\lvert f_+(0)\rvert$	m_+(GeV)
E687 [16]	0.71 ± 0.04	1.87 ± 0.13
CLEO [18]	0.77 ± 0.04	2.00 ± 0.22

From an experimental point of view, the data are usually fitted to one-pole form factors:

$$f_\pm(q^2) = \frac{f_\pm(0)}{1 - \dfrac{q^2}{m_\pm^2}}, \tag{43}$$

though due to the m_ℓ-suppression pointed out in our discussion related with (27) $f_-(q^2)$ is largely unknown. Other parameterizations for $f_+(q^2)$ are also possible. In particular, and due to pioneering modelizations [9], the exponential behavior $f_+(q^2) = f_+(0)\exp(\alpha q^2)$ has also been fitted to the data. Nevertheless in the available range of energies it is not possible to distinguish both parameterizations. However from experiment one gets $\alpha = (0.29 \pm 0.07)\,\mathrm{GeV}^{-2} > 0$ [18] and, therefore, the asymptotic behavior of this latter parameterization is disastrous in accordance with our discussion in Sect. 3. Surely the exponential form factor is not consistent with QCD. Moreover, notice that a one-pole form factor only for $f_-(q^2)$, as in (43), is not allowed (unless $f_+(q^2) = 0$) because $F_0(q^2)$ in (35) would drive a $J^P = 1^-$ transition, through the pole of the vector resonance, which is forbidden for that form factor.

Hence in $f_+(q^2)$ there are two parameters to fit: $f_+(0)$ and the pole mass m_+. The experimental figures are collected in Table 1. From our result in (40) we see that

$$f_+(0)[D,P] = \Omega[D,P]. \tag{44}$$

Hence, from experiment, $|\Omega[D^0,K^-]| \simeq 0.75$ in excellent agreement with the sum rules expectations [35]. On the other side the obtained values of m_+ are consistent with the experimental value of $m_{D_s^V} = m_{D_s^*} = 2.1124 \pm 0.0007\,\mathrm{GeV}$, which is the one appearing in our form factor.

The study of the ratio $\mathrm{Br}(D^0 \to \pi^-\ell^+\nu_\ell)/\mathrm{Br}(D^0 \to K^-\ell^+\nu_\ell)$ provides information on the difference between the $f_+(q^2)$ form factors with K or π in the final state. Experimental figures are given in Table 2. From our results we predict

$$\frac{|f_+(0)[D^0,\pi^-]|}{|f_+(0)[D^0,K^-]|} = \frac{m_{D_s^V}^2}{m_{D^V}^2} \simeq 1.05, \tag{45}$$

if we take, from [12], $m_{D_s^V} = m_{D_s^*}$ and $m_{D^V} = m_{D^*(2010)^\pm}$.

The structure provided by our approach for $f_-(q^2)$ in (40) is much more complex. We have a two-pole structure that would be very interesting to explore phenomenologically. Unfortunately, to our knowledge, the only known experimental result on $f_-(q^2)$ is provided by the E687 Collaboration [16] giving

$$\frac{f_-(0)[D^0,K^-]}{f_+(0)[D^0,K^-]} = -1.3 \pm^{3.6}_{3.4}, \tag{46}$$

Table 2. Experimental values for the ratio $|f_+(0)[D^0,\pi^-]|/|f_+(0)[D^0,K^-]|$. We have used that $(|V_{cd}|/|V_{cs}|)^2 = 0.051 \pm 0.001$

Experiment	$\dfrac{\lvert f_+(0)[D^0,\pi^-]\rvert}{\lvert f_+(0)[D^0,K^-]\rvert}$
E687 [15]	1.00 ± 0.12
CLEO [20]	1.01 ± 0.21

still compatible with zero, to compare with our result:

$$\frac{f_-(0)[D^0,K^-]}{f_+(0)[D^0,K^-]} = -\frac{m_{D^0}^2 - m_{K^-}^2}{m_{D_s^V}^2} + \frac{\Lambda[D^0,K^-]}{\Omega[D^0,K^-]}. \tag{47}$$

In our prediction the first term gives $(m_{K^-}^2 - m_{D^0}^2)/m_{D_s^V}^2 \simeq -0.72$, agreeing in sign and size with the central value in the experimental determination. If we take $\Lambda[D^0,K^-]_{FACT}$ we would have a prediction for the ratio in (47) in terms of masses of resonances and pseudoscalars. Unfortunately the unknown scalar charmed meson mass is also involved. Although we know very little about $\Lambda[D,K]$ from the phenomenology, determinations of $f_\pm(0)$ within a sum rule approach provide information on Λ. With the results of [35] we find $\Lambda[D^0,K^-] = -0.05 \pm 0.11$ and $\Lambda[D^0,\pi^-] = -0.03 \pm 0.12$, which is hence compatible with zero. Accordingly the ratio in (47) is very well approximated by the first term only and, in addition, we can conclude that the contribution of the scalar resonances to $f_-(q^2)$ in (40) should be tiny. Moreover notice that the sum rules predictions are at odds with $\Lambda[D,P]_{FACT}$ unless the lightest scalar charmed resonance has a very large mass.

In conclusion much more work is needed on the experimental side to be able to compare our results with the phenomenology. The spectrum of the semileptonic decays of the charmed mesons should be measured with good accuracy in order for us to confirm the structure of $f_+(q^2)$ and find out if the peculiar two-pole feature of QCD and saturation by the resonances driven by $f_-(q^2)$ are confirmed. With these analyses we could take a serious step forward in the determination and comprehension of the effective action of QCD in the charm energy region.

5 Other semileptonic decays

The effective action $\mathcal{S}_{\mathrm{eff}}$ in (18) allows us to evaluate all semileptonic $D_{\ell 3}$ and $D_{\ell 4}$ decays. A thorough phenomenological study of them would make well known the couplings of the operators in $\mathcal{L}_{\mathrm{eff}}$ that determine their strength. In this first paper we have addressed the study of the simplest processes $D \to P\ell^+\nu_\ell$ with the conclusions pointed out in Sects. 3 and 4. We stress here the interrelation between the couplings and other processes.

Decay constants of the mesons parameterize the transition from the meson to the hadronic vacuum. While there is a reasonably adequate knowledge on the D decay constant F_D [12] from $D_{\ell 2}$ decays, the phenomenological

Table 3. Couplings or combinations of couplings from $\mathcal{L}_{\rm eff}$ appearing in the form factors of semileptonic decays of the charmed mesons. As in the main text P is short for a light pseudoscalar meson and V is short for a light vector meson

Processes	Couplings
$D \to P\ell^+\nu_\ell$	F_D, α_1, $\beta_4 F_{D^V}$, $\beta_2 F_{D^S}$, $\beta_3 F_{D^S}$
$D \to PP\ell^+\nu_\ell$	F_D, α_1, $\beta_4 F_{D^V}$
$D \to V\ell^+\nu_\ell$	α_2, $\beta_1 F_D$, $\beta_\varepsilon F_{D^V}$, $\beta_5 F_{D^A}$

determination of the decay constants of the resonances F_{D^S}, F_{D^V} and F_{D^A} involves electroweak decays (such as $D_{\rm R} \to \ell^+\nu_\ell$,...) that are tiny against the strong dominating processes. Therefore their experimental evaluation is out of question. In addition β_ε, β_1 and β_5 only appear in off-shell strong vertices. The strong couplings β_2, β_3 and β_4 in $\mathcal{L}_{\rm eff}$ could be determined from on-shell strong processes. Although the first two involve still unobserved scalar charmed resonances, the β_4 coupling, which drives $D^* \to D\pi$, can be obtained through the recent observation of this decay [36]. From this width we get $|\beta_4| = 0.58 \pm 0.07$. Notice that β_4 is involved in the determination of $f_+(0)$ (see (41) and (44)); however, we do not know the value of the decay constant of the vector charmed resonances, F_{D^V}, and consequently we cannot predict $f_+(0)$ in a model-independent way. Reversely, using its experimental value we can determine $|F_{D^V}| \sim 240\,\text{MeV}$.

The role of the phenomenology of semileptonic processes to get information on these couplings is relevant. In these decays we usually have amplitudes that involve one coupling, like the vertex in Fig. 1a, or the product of two couplings, as the two connected vertices in Fig. 1b. A close look at the $\mathcal{L}_D$, $\mathcal{L}_{D^S}$, $\mathcal{L}_{D^V}$ and $\mathcal{L}_{D^A}$ Lagrangians shows the couplings relevant for the different processes. We collect them in Table 3. Notice that the $D \to V\ell^+\nu_\ell$ processes also contribute to the $D_{\ell 4}$ decays through the strong conversion $V \to PP$ driven by $\mathcal{L}_{\rm R\chi PT}$ in (18), of which the couplings are rather well known.

The foreseen good prospects on the experimental side for the near future, together with the QCD constraints from the dynamical behavior in the asymptotic limit (not taken into account when writing Table 3), that also should extend properly to the $D \to V\ell^+\nu_\ell$ and $D_{\ell 4}$ processes, should enable us to determine reasonably well the effective action of QCD in this energy regime.

6 Comparison with the heavy quark mass expansion

An alternative approach based also on a phenomenological Lagrangian that tries to implement both HQET [2] and chiral symmetry [27] has been employed during the last years [3,4] in the study of heavy $\to$ light semileptonic processes. This is a rigorous and systematic procedure that deals with the construction of an effective action of QCD through the constraints of heavy quark and chiral symmetries and that inherits from HQET the perturbative expansion in inverse powers of the heavy quark mass, typically m_q/M_Q and $\Lambda_{\rm QCD}/M_Q$, where m_q and M_Q are the masses of light and heavy quarks, respectively.

This feature has several consequences. On the one side the fast convergence of the perturbative expansion in the study of B meson decays, because of the high mass of the b quark, does not apply so clearly in the case of the D meson decays. Moreover although the effective action is very simple in the $M_Q \to \infty$ limit, where the nice property of the relation of the B and D processes arises, it becomes rather cumbersome when next-to-leading terms in the mass expansion are included, consequently losing predictability, unless some modelization hypotheses are assumed [5]. On the other side, heavy-quark symmetry relations are useful if the recoiling light constituents can only probe distances that are large compared with $1/M_Q$. This condition is equivalent to the statement $(v \cdot v' - 1) \ll M_Q/\Lambda_{\rm QCD}$ or $q^2 \simeq q^2_{\rm max} = (m_D - m_P)^2$ in $D \to P\ell^+\nu_\ell$, where v and v' are the four-velocities of the initial and final hadrons. Hence in this framework one evaluates $f_\pm(q^2_{\rm max})$, a particular analytic continuation for the form factors (usually a monopole structure given by vector meson dominance) is assumed and, in consequence, a prediction for $f_\pm(0)$ is given. It is necessary to emphasize that the prediction of the form factors, given by the heavy quark mass expansion, at $q^2 \neq q^2_{\rm max}$ includes input from outside the perturbative treatment.

The effective theory framework that we propose in this article, on the other hand, relies on well-known aspects of the underlying QCD theory. We skip the heavy quark mass expansion by applying the well-known procedure of constructing a phenomenological Lagrangian [22] on the basis of chiral symmetry (for the light flavors) and considering the charm flavored mesons as matter fields in specific $SU(3)$ representations that provide the interaction. The phenomenological Lagrangian acquires the specific features of QCD by imposing the high energy behavior on the form factors, a procedure that constrains the couplings. This is an essential step in the construction of the effective action in order to improve our Lagrangian with another model-independent tool that facilitates the matching at higher energies. This whole methodology is analogous to the one used in the resonance chiral theory. In addition the dynamical structure of the form factors does not rely on assumed analytic continuations but on the prediction of QCD in the limit of a large number of colors ($N_C \to \infty$). As emphasized above this limit establishes the role of single resonances in the Green functions and, consequently, in our form factors. Notice that the procedure we are presenting may be extended systematically by including next-to-leading corrections in the $1/N_C$ expansion though, as in the heavy quark mass expansion, one needs to perform the construction of the action at one-loop level.

A complete comparison between our results for the semileptonic form factors $f_\pm(q^2)$ and those of the heavy mass expansion at leading order (which we take from [4] for definiteness) is not feasible because of the different input included in their construction. The main difference

arises from the high energy constraints on our effective action. These have no clear meaning in the heavy quark mass expansion when one perturbates around the heavy quark mass. On the other side our results for the form factors (40) include the contributions of scalar resonances that have not been taken into account in the heavy mass expansion approach. However it is easy to see that, switching off these scalar contributions and performing a heavy meson mass expansion on our results in (40), we recover the features of the heavy quark mass expansion results in [4]:

$$f_+(q^2) + f_-(q^2) \simeq 2\frac{\Delta}{M_V} f_+(q^2),$$
$$f_+(q^2) - f_-(q^2) \simeq 2 f_+(q^2), \qquad (48)$$

where $\Delta = M_V - m_D$. Moreover, in this limit, our results for $f_+(q^2)$ coincide with those of that reference provided that $|\beta_4 F_{D^V}/2| = |gF_D|$, where g drives the $D^* \to D\pi$ decay in HQET. From this latter process we see that $|\beta_4| = |g|$ and, in addition, with our definition of the decay constants the heavy quark spin symmetry demands that $F_{D^V} = 2F_D$. Hence the consistency of our prediction for the form factors $f_\pm(q^2)$ with the heavy quark mass limit is exact. However we stress that our results include the mass corrections to that limit.

In Sect. 5 we got $|F_{D^V}| \sim 240\,\mathrm{MeV}$. The heavy quark spin symmetry demands that $F_D = F_{D^V}/2 \sim 120\,\mathrm{MeV}$ and, experimentally, the value of this decay constant is still rather uncertain, $F_D = 212\,^{+139}_{-109}$ MeV [12]. Notice however that, as emphasized in [37], the spin-symmetry breaking effects in the charmed sector could be as large as 50%.

7 Conclusions

The study of the form factors of the QCD currents provides all-important information on the relevant effective action of the underlying theory. Semileptonic decays of the mesons are the main tool to analyze charged currents and, while B and K decays have received very much attention, D decays, due to their position in the energy spectra, lack a definite and sound framework in which to perform this task.

We have proposed a model-independent scheme that relies on the use of phenomenological Lagrangians generated through the symmetries of QCD and the dynamics of its $N_C \to \infty$ limit. In this scheme the three lightest flavors are introduced following the guide of chiral symmetry while the charmed mesons appear as matter fields, following [21,22]. The procedure is analogous to the construction of the resonance chiral theory [24]. Hence we arrive at an effective field theory where the structure of the operators is driven by the symmetries and their couplings are unknown. In addition, the QCD-ruled asymptotic behavior of the form factors imposes several constraints on those couplings. In this framework, we have computed the form factors in the semileptonic $D \to P\ell^+\nu_\ell$ processes at leading order in the $1/N_C$ expansion and we end with the parameterizations in (40) that are our main result. It is necessary to emphasize that this approach is different from the one followed in [3,4] which relies on the heaviness of the charmed quark, while here this consideration, with its possible misconceptions in the charmed case, does not appear. Moreover we do not need to assume a particular structure for the analytic continuation of the form factors because we rely on the dynamics driven by the $N_C \to \infty$ limit of QCD.

The experimental situation in $D \to P\ell^+\nu_\ell$ is rather poor though it is foreseen to be upgraded in the near future. While our result for $f_+(q^2)$ is consistent with the experimental analysis, we would consider it very interesting that, through the $D \to P\mu^+\nu_\mu$ processes, something could be said about the $f_-(q^2)$ form factor, where we have concluded that a two-pole structure is predicted in our framework. The parameterization we propose to analyze the experimental data by is then

$$f_+(q^2) = \frac{a_V}{1 - \dfrac{q^2}{M_V^2}},$$
$$f_-(q^2) = \frac{b_V}{1 - \dfrac{q^2}{M_V^2}} + \frac{b_S}{1 - \dfrac{q^2}{M_S^2}}. \qquad (49)$$

At present a_V, b_V (which is proportional to a_V according to our prediction in (40)) and M_V are rather well known. However nothing can be said about the size of b_S or M_S (scalar resonances with charm have not been observed) and, consequently, this should be an important task for future research in this field. If one uses the definition of the form factors in (35) instead, $F_0(q^2)$ should show, in addition to the one-pole structure induced by the scalar resonances, a non-negligible local piece acting as a background. Once all these observables are measured we will be able to constrain the effective action by better determining the strength of its operators.

A complementary study of the form factors in $D \to V\ell^+\nu_\ell$ within the effective action of QCD proposed in this paper is under way [25].

Finally we have shown that, while it is not possible to apply QCD directly to the study of these hadronic processes, it is definitely feasible to extract model-independent information on the form factors of the QCD currents by exploiting and implementing the known features of the underlying theory, such as the symmetries or dynamic behavior, providing a compelling framework to work with.

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